CogSci 109 Fall 2006 Math Review notes

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0.1 Description

Here you will find some useful mathematical techniques with an example or two as relevant. This document will be updated as the quarter progresses.

0.2 Calculus

Limit laws

from Stewart, 3rd edition, page 61

Suppose c is a constant and the limits

$lim_{x->a}f(x)$ and $lim_{x->a}g(x)$

exist. Then

1. $lim_{x->a}[f(x) + g(x)] = [lim_{x->a}f(x)] + [lim_{x->a}g(x)]$
2. $lim_{x->a}[f(x) - g(x)] = [lim_{x->a}f(x)] - [lim_{x->a}g(x)]$
3. $lim_{x->a}[cf(x)] = c[lim_{x->a}f(x)]$
4. $lim_{x->a}[f(x) * g(x)] = [lim_{x->a}f(x)] * [lim_{x->a}g(x)]$
5. $lim_{x->a}[\frac{(x)}{g(x)}] = \frac{[lim_{x->a}f(x)]}{[lim_{x->a}g(x)]}$ if $[lim_{x->a}g(x)] \neq 0$
6. $lim_{x->a}[f(x)]^n = [[lim_{x->a}f(x)]^n$ where n is a positive integer
7. $lim_{x->a}c = c$

8. $lim_{x->a}x = a$
9. $lim_{x->a}x^n = a^n$ where n is a positive integer
10. $lim_{x->a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer (If n is even, we assume that a¿0.)
11. $lim_{x->a} \sqrt[n]{f(x)} = \sqrt[n]{lim_{x->a}f(x)}$ where n is a positive integer (If n is even, we assume that $lim_{x->a}f(x) > 0$

Definitions

from Stewart, 3rd edition, page 82

1. A function f is continuous at a number a if

   $lim_{x->a}f(x) = f(a)$

2. a function f is continuous from the right at a number a if

   $lim_{x->a^+}f(x) = f(a)$

   and f is continuous from the left at a if

   $lim_{x->a^-}f(x) = f(a)$

3. A function f is continuous on an interval if it is continuous at every number in the interval. (At an endpoint of the interval we understand continuous to mean continuous from the right or continuous from the left.)

Example

Show that the function f(x)=1-x is continuous on the interval [-1,1].

Solution: If -1¡a¡1, then using the limit laws, we have

$lim_{x->a}f(x) = lim_{x->a}(1 - x)$

$= 1 - lim_{x->a}(x)$

$= 1 - a = f(a)$

Thus by definition 1, f is continuous at a if $-1 < a < 1$. We must also calculate the right-hand limit at -1 and the left hand limit at 1.

$lim_{x->-1^+} f(x) = lim_{x->-1^+}(1-x)$

$= 1 - lim_{x->-1^+}(x)$

$= 1 - (-1)$

$= 2 = f(-1)$

So f is continuous from the right at -1. Similarly,

$lim_{x->1^-} f(x) = lim_{x->1^-}(1-x)$

$= 1 - lim_{x->1^-}(x)$

$= 1 - (1)$

$= 0 = f(1)$

So f is continuous from the left at 1. Therefore, according to Definition 3, f is continuous on [-1,1] (recall that the "[.]" brackets mean including those points).

Derivatives

$\frac{d}{dx}(c) = 0$ (where c is some constant)

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(sin(x)) = cos(x)$

$\frac{d}{dx}(cos(x)) = -sin(x)$

$\frac{d}{dx}(tan(x)) = sec^2(x)$

$\frac{d}{dx}(csc(x)) = -csc(x) * cot(x)$

$\frac{d}{dx}(sec(x)) = sec(x) * tan(x)$

$\frac{d}{dx}(cot(x)) = -csc^2(x)$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(e^u) = e^u * \frac{du}{dx}$ (where u is an arbitrary function)

$\frac{d}{dx}(log_{10}(x)) = \frac{1}{x}$

$(cf)' = c(f)'$ (where c is a constant)

$(f-g)' = f' - g'$

$(f+g)' = f' + g'$

$(fg)' = f'g + g'f$

$(\frac{f}{g})' = \frac{f'g - g'f}{g^2}$

$\frac{d}{dx}(a^x) = a^x * ln(a)$ (where a is some constant)