Hints for Assignment 2 : Hints about limits and continuity

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1 Description

The first thing to do is review the first chapter of your calculus book, especially the Stewart Early Transcendentals book.

Having glanced over that, you may have already reminded yourself of what you need. But if you are still a bit foggy, here's a quick group of useful rules about limits and an example problem which is similar to the homework:

1.1 Limit laws

from Stewart, 3rd edition, page 61

Suppose c is a constant and the limits

$lim_{x->a}f(x)$ and $lim_{x->a}g(x)$

exist. Then

1. $lim_{x->a}[f(x) + g(x)] = [lim_{x->a}f(x)] + [lim_{x->a}g(x)]$
2. $lim_{x->a}[f(x) - g(x)] = [lim_{x->a}f(x)] - [lim_{x->a}g(x)]$
3. $lim_{x->a}[cf(x)] = c[lim_{x->a}f(x)]$
4. $lim_{x->a}[f(x) * g(x)] = [lim_{x->a}f(x)] * [lim_{x->a}g(x)]$

5. $lim_{x->a}[\frac{(x)}{g(x)}] = \frac{[lim_{x->a}f(x)]}{[lim_{x->a}g(x)]}$ if $[lim_{x->a}g(x)] \neq 0$
6. $lim_{x->a}[f(x)]^n = [[lim_{x->a}f(x)]^n$ where n is a positive integer
7. $lim_{x->a}c = c$
8. $lim_{x->a}x = a$
9. $lim_{x->a}x^n = a^n$ where n is a positive integer
10. $lim_{x->a}\sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer (If n is even, we assume that a¿0.)
11. $lim_{x->a}\sqrt[n]{f(x)} = \sqrt[n]{lim_{x->a}f(x)}$ where n is a positive integer (If n is even, we assume that $lim_{x->a}f(x) > 0$

1.2 Definitions

from Stewart, 3rd edition, page 82

1. A function f is continuous at a number a if

   $lim_{x->a}f(x) = f(a)$

2. a function f is continuous from the right at a number a if

   $lim_{x->a^+}f(x) = f(a)$

   and f is continuous from the left at a if

   $lim_{x->a^-}f(x) = f(a)$

3. A function f is continuous on an interval if it is continuous at every number in the interval. (At an endpoint of the interval we understand continuous to mean continuous from the right or continuous from the left.)

1.3 Example

Show that the function f(x)=1-x is continuous on the interval [-1,1].

Solution: If $-1 < a < 1$, then using the limit laws, we have

$lim_{x->a}f(x) = lim_{x->a}(1 - x)$

$= 1 - lim_{x->a}(x)$

$= 1 - a = f(a)$

Thus by definition 1, f is continuous at a if $-1 < a < 1$. We must also calculate the right-hand limit at -1 and the left hand limit at 1.

$lim_{x->-1^+} f(x) = lim_{x->-1^+}(1 - x)$

$= 1 - lim_{x->-1^+}(x)$

$= 1 - (-1)$

$= 2 = f(-1)$

So f is continuous from the right at -1. Similarly,

$lim_{x->1^-} f(x) = lim_{x->1^-}(1 - x)$

$= 1 - lim_{x->1^-}(x)$

$= 1 - (1)$

$= 0 = f(1)$

So f is continuous from the left at 1. Therefore, according to Definition 3, f is continuous on [-1,1] (recall that the "[.]" brackets mean including those points).