CogSci 109: Lecture 24

Wed Dec 4, 2007

Multilayer artificial neural networks, self-organizing maps, and examples, and applications (III)

Outline for today

- Announcements
- Homework announcement
- Practice final up later
- Review session day?
- Grades online updated
  - **Hw2 issues should have been resolved**

Outline for today II

- About the final
  - **Takehome portion like a homework, worth 200 points**
    - Due Saturday at 12 midnight at the end of finals week
  - **In class portion multiple choice, like the midterm mult choice, but more questions**
  - **Practice final posted later this week, probably Wed**
    - With solutions
  - **Cumulative, but will focus on material since midterm**
  - **Bring 3 double sided pages of notes, handwritten**
  - **Bring calculator**
  - **Bring red scantron**
  - **Bring plenty of pencils and erasers**

Outline for today (III)

- Review of last time
  - **Potential issues with training networks**
    - Overfitting and generalization
    - **nnd11gn**
  - **Methods of dealing with these issues**
- Unsupervised learning and associative memory
  - **Hopfield network**
  - **Binary network learning rule**
  - **Extension to continuous forms**
  - **Stability of memories**
  - **Brain damage**
  - **failure**

Potential issues to deal with when training neural networks

- ***Overfitting*** **- the state in which a model that has too many parameters (degrees of freedom) adapts too well to training data, fitting noise**
  - The system then does not respond properly to new input data of the same class
- ***Generalization*** **- ability of a learning system to correctly map new inputs that were not previously used in the training phase**
- **We want to reduce overfitting and increase generalization of our fits**

Techniques to Prevent Overfitting

- Regularization
  - **Reduction of hidden units**
    - Only fit simpler functions
  - **Weight decay**
- Early stopping
  - **Using validation sets**
- Bayesian regularization
  - **(see the MacKay Book)**

Technique 1: Reduce number of layers to prevent overfitting

- *Note: Remember that overfitting is a problem when fitting many parameters to small amounts of data*
  - ***Infinite data would be then no problem***
- Simplify the function you are fitting by reducing the number of network hidden layers - similar to using a lower degree polynomial to fit data
  - **Limits the capability of your network**
- But ahead of time we may not know the complexity of the function we want to fit, so how do we deal with this?

Technique 2: Regularization to prevent overfitting

- ***Regularization*** - adding a penalty to the usual error function to encourage smoothness

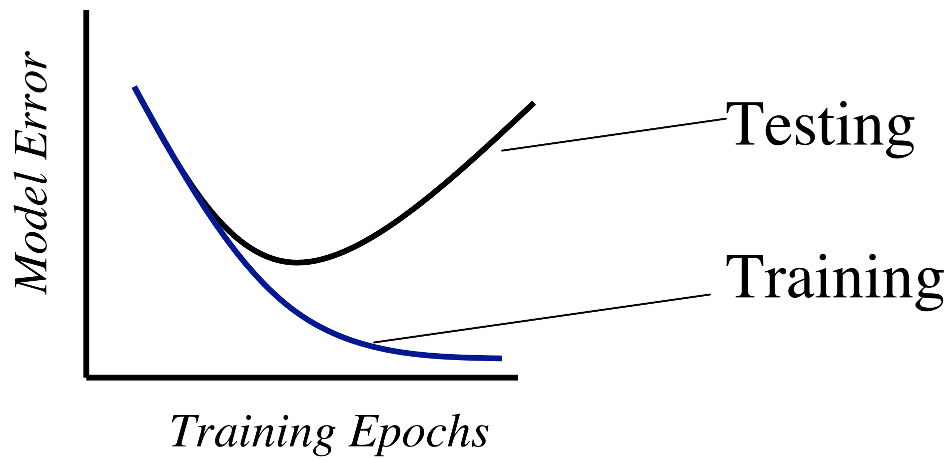
$$\mathrm{E}_{\mathrm{new}} = \mathrm{E} + \nu * \omega$$

- Here ν is the regularization parameter and ω is the smoothness penalty

- Weight decay sets $$\omega = \frac{1}{2}\sum_i w_i^{\,2}$$
  - **Note that when you then take the partial derivative of E_{new} with respect to a weight the update rule will now include a term that is -w_i.**
  - **This will encourage the weights to decay to zero (hence the name)**

Technique 3: Early stopping to prevent overfitting

- Start the weights very small
  - **Then the neural network starts by behaving fairly linearly**
  - **The weights gradually increase to handle nonlinearities**
- Split the data into a **validation set** and a **training set**
  - **Use the *training set* to adjust the weights**
  - **Use the *validation set* to compute model error**
  - **As the fit improves the error will decrease, when the error starts to increase again, you are fitting the noise in the training set**



Technique 4: Bayesian regularization to prevent overfitting

- The Bayesian neural network formalism of David MacKay and Radford Neal, considers neural networks not as single networks but as distributions over weights (and biases)
- The output of a trained network is thus not the result of applying one set of weights but an average over the outputs from the distribution.
- This can be computationally expensive but MacKay and Neal have developed approximations and the approach leads to automatic regularization that is very effective.

More training issues

- Improvements on gradient descent
  - **Gradient descent with momentum**
  - ***Conjugate gradient***
  - **Variable learning rate**
  - **For nonquadratic functions, minimization (ie Nelder Mead, golden section line search, Brent's method, etc - See numerical methods book)**
    - Demos:
      - **nnd12sd1**
      - **nnd12sd2**
      - **nnd12mo**
      - **nnd12vl**
      - **nnd12ls**
      - **nnd12cg**

Unsupervised learning for associative memory

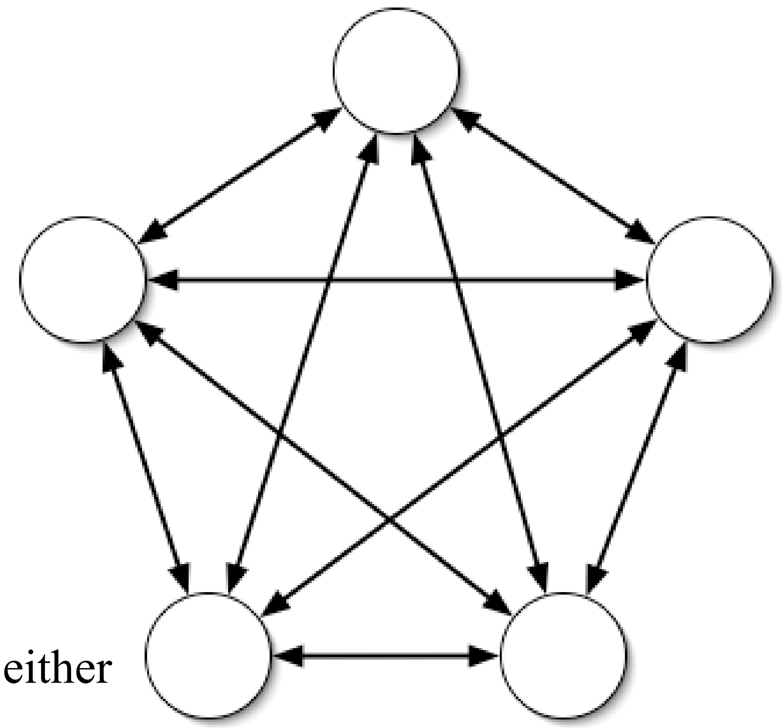
- Hebbian learning (Hebb 1949)
- The weights of neurons whos activities are positively correlated are increased:

$$\frac{dw_{ij}}{dt} \sim Correlation(x_i, x_j)$$

- So when stimulus m is present, the activity of neuron m increases
- Neuron n is associated with another stimulus n
- If these two stimuli co-occur in the environment, the Hebbian learning rule will increase the weights w_{nm} and w_{mn}
  - **Now when stimulus n appears later alone, the positive weight from n->m will cause neuron m to be also activated**

A Network example - Associative Memory

- Associative memory sample
  - **(Yellow)--(banana smell)**
- What is a binary Hopfield network?
  - **Weights are constrained to be**
    - Symmetric $w_{kp} = w_{pk}$
    - Bidirectional
    - No self connections (w_ii = 0)
  - **Activity rule** $x(a) = \Theta(a) \equiv \begin{cases} 1 & a \geq 0 \\ -1 & a < 0 \end{cases}$



    - We need to specify the order of updates as either
      - **Synchronous**

$$a_k = \sum_j w_{kj} x_j$$

$$x_k = \Theta(a_k)$$

      - **Asynchronous - each neuron sequentially (either fixed or random order) computes its activation then updates its output state and weights**

Binary Network learning rule

- Learning rule
  - **The problem - make a set of memories $\{\mathbf{x}^{(n)}\}$ stable states of the network's activity rule**
    - Each memory is a binary pattern $x_i \in \{-1, 1\}$
    - Setting the weights is done according to Hebb's rule:

$$w_{ij} = \eta \sum_n x_i^{(n)} x_j^{(n)}$$

    - We may set eta to prevent a particular weight from growing with N:

$$\eta = 1/N$$

Associative memory example

Desired memories:

moscow------russia
lima----------peru
london-----england
tokyo--------japan
edinburgh-scotland
ottawa------canada
oslo--------norway
stockholm---sweden
paris-------france

- Pattern completion

moscow---:::::::::: ⟹ moscow------russia
::::::::::--canada ⟹ ottawa------canada

- Error correction

otowa-------canada ⟹ ottawa------canada
egindurrh-sxotland ⟹ edinburgh-scotland

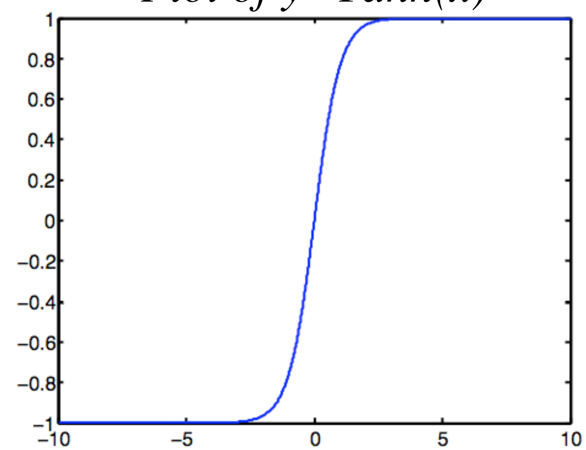
Continuous form of the Hopfield network

- Similar rules, but instead of binary states, we have continuous states from (-1,1)

$$a_i = \sum_j w_{ij} x_j$$

$$x_i = \tanh(a_i)$$

Plot of y=Tanh(x)



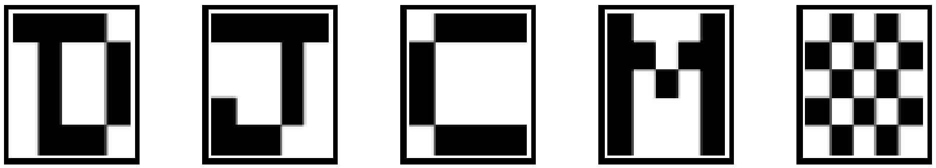
- Eta becomes more important

Stability of memories

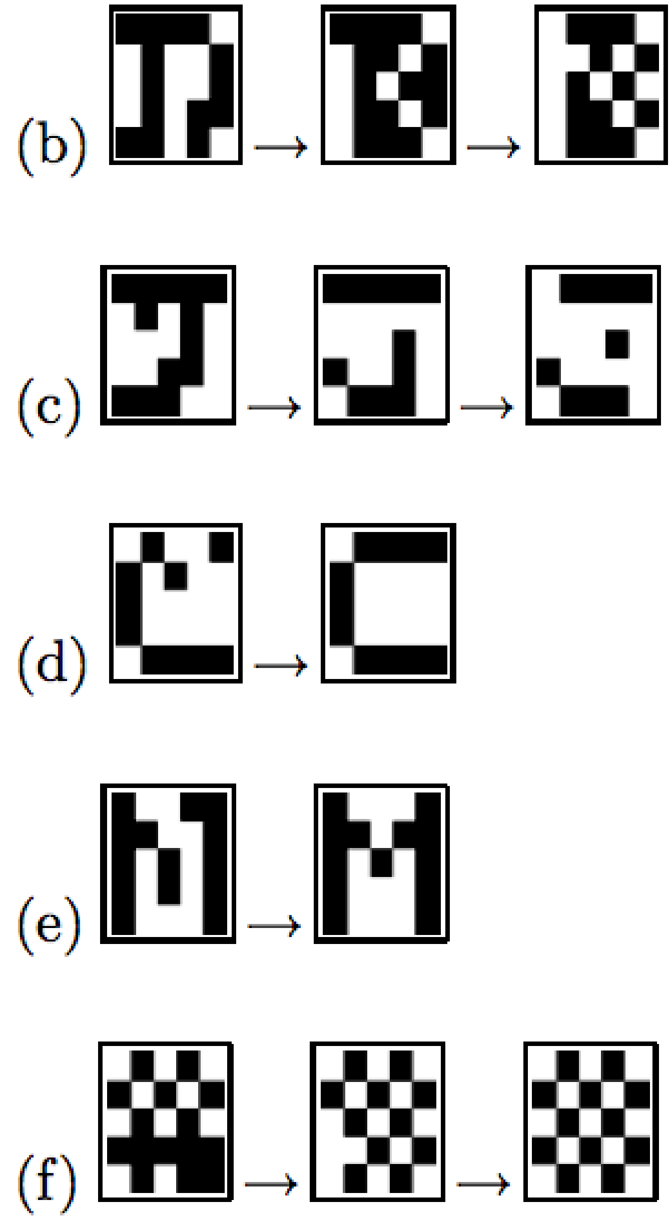
- Lyapunov functions
  - **If you can show that a lypapunov function exists for an ANN, then it's dynamics converge rather than diverge**
  - **Look up lyapunov functions for more info, there is not time to cover them here**

Brain damage (p. 511 in MacKay) - delete 26 weights, still converges

(a)



.	-1	1	-1	1	x	x	-3	3	x	x	-1	1	-1	x	-1	1	-3	x	1	3	-1	1	x	-1
-1	.	3	5	-1	-1	-3	-1	-3	-1	-3	1	x	1	-3	1	-1	-1	-1	-1	-3	5	3	3	-3
1	3	.	3	1	-3	-1	x	-1	-3	-1	-1	x	-1	-1	-1	1	-3	1	-3	-1	3	5	1	-1
-1	5	3	.	-1	-1	-3	-1	-3	-1	-3	1	-5	1	-3	1	-1	-1	-1	-1	-3	5	x	3	-3
1	-1	1	-1	.	1	-1	-3	x	x	3	-5	1	-1	-1	3	x	-3	1	-3	3	-1	1	-3	3
x	-1	-3	-1	1	.	-1	1	-1	1	3	-1	1	-1	-1	3	-3	1	x	1	x	-1	-3	1	3
x	-3	-1	-3	-1	-1	.	-1	1	3	1	1	3	-3	5	-3	3	-1	-1	x	1	-3	-1	-1	1
-3	-1	x	-1	-3	1	-1	.	-1	1	-1	3	1	x	-1	-1	1	5	1	1	-1	x	-3	1	-1
3	-3	-1	-3	x	-1	1	-1	.	-1	1	-3	3	1	1	1	-1	-1	3	-1	5	-3	-1	x	1
x	-1	-3	-1	x	1	3	1	-1	.	-1	3	1	-1	3	-1	x	1	-3	5	-1	-1	-3	1	-1
x	-3	-1	-3	3	3	1	-1	1	-1	.	-3	3	-3	1	1	-1	-1	-1	-1	1	-3	-1	-1	5
-1	1	-1	1	-5	-1	1	3	-3	3	-3	.	-1	1	1	-3	3	x	-1	3	-3	1	-1	3	-3
1	x	x	-5	1	1	3	1	3	1	3	-1	.	-1	3	-1	1	1	1	1	3	-5	-3	-3	3
-1	1	-1	1	-1	-1	-3	x	1	-1	-3	1	-1	.	x	1	-1	3	3	-1	1	1	-1	-1	-3
x	-3	-1	-3	-1	-1	5	-1	1	3	1	1	3	x	.	x	3	-1	-1	3	1	-3	-1	-1	1
-1	1	-1	1	3	3	-3	-1	1	-1	1	-3	-1	1	x	.	-5	-1	-1	-1	1	1	-1	-1	1
1	-1	1	-1	x	-3	3	1	-1	x	-1	3	1	-1	3	-5	.	1	1	1	-1	-1	1	1	-1
-3	-1	-3	-1	-3	1	-1	5	-1	1	-1	x	1	3	-1	-1	1	.	1	1	-1	-1	-3	1	-1
x	-1	1	-1	1	x	-1	1	3	-3	-1	-1	1	3	-1	-1	1	1	.	-3	3	-1	1	-3	-1
1	-1	-3	-1	-3	1	x	1	-1	5	-1	3	1	-1	3	-1	1	1	-3	.	x	-1	-3	1	-1
3	-3	-1	-3	3	x	1	-1	5	-1	1	-3	3	1	1	1	-1	-1	3	x	.	-3	-1	-5	1
-1	5	3	5	-1	-1	-3	x	-3	-1	-3	1	-5	1	-3	1	-1	-1	-1	-1	-3	.	3	x	-3
1	3	5	x	1	-3	-1	-3	-1	-3	-1	-1	-3	-1	-1	-1	1	-3	1	-3	-1	3	.	1	-1
x	3	1	3	-3	1	-1	1	x	1	-1	3	-3	-1	-1	-1	1	1	-3	1	-5	x	1	.	-1
-1	-3	-1	-3	3	3	1	-1	1	-1	5	-3	3	-3	1	1	-1	-1	-1	-1	1	-3	-1	-1	.



Failures of ANN's

- Stability of memories is an issue to be considered
- For failure mode analysis (where hopfield networks fail to correctly restore memories), see MacKay Chapter 42