CogSci 109: Lecture 18

Monday Nov. 19, 2007

Error analysis examples, introduction to function minimization

Outline for today

- Announcements
- Error analysis examples
- Introduction to minimization and optimization
  - **What is optimization?**
  - **What is minimization?**
  - **Fminsearch - definition and algorithm**
  - **examples**

Announcements

- Homework
- Readings/handouts
- Friday is Thanksgiving!!! No class or section (no section Thurs either)
- Thursday and Friday sections please try to make the Wed sections if possible

Quick review of error analysis methods

- There are many ways to estimate errors, here are a couple of common ones
  - **To get a single # - can use various norms**
    - 2-norm

$$\|e\|_2 = \sqrt{\sum_i (y_i - \hat{y}_i)^2}$$

    - Mean-squared-error

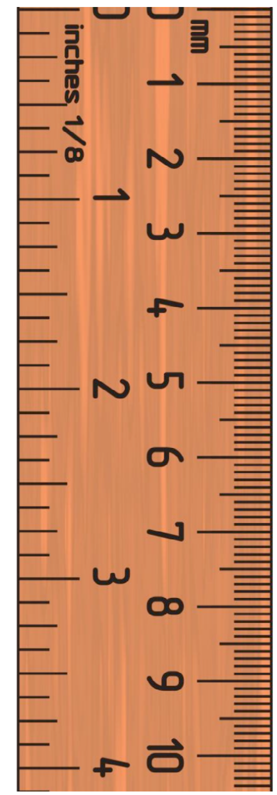
$$MSE = \frac{1}{N}\sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

  - **Curve - simple error (for a time dependent signal y(t))**

$$e(t) = y(t) - \hat{y}(t)$$

  - **Curve - prediction error**

$$e_p(t) = y(t) - \hat{y}(t \mid t-1)$$



Demo -

Second demo

What can we do with this idea of error?

- We now can quantify differences between model and reality
- Gives us a criterion for choosing and creating models
- What do I mean by this?
  - **Let me pose the question - *How can we fit a model which is nonlinear in the parameters?***
    - Least squares won't work!
    - Could linearize for the parameters…but what about cases where that is too difficult?

Optimization for regression problems which are nonlinear in the parameters

- **Optimization -** the study of problems where the goal is to minimize or maximize a function by strategically choosing values for a set of variables
  - **This is typically an iterative process, though in many cases one can solve for the optimal point of the function**

Optimization is a popular way to study the human brain, behavior and computation

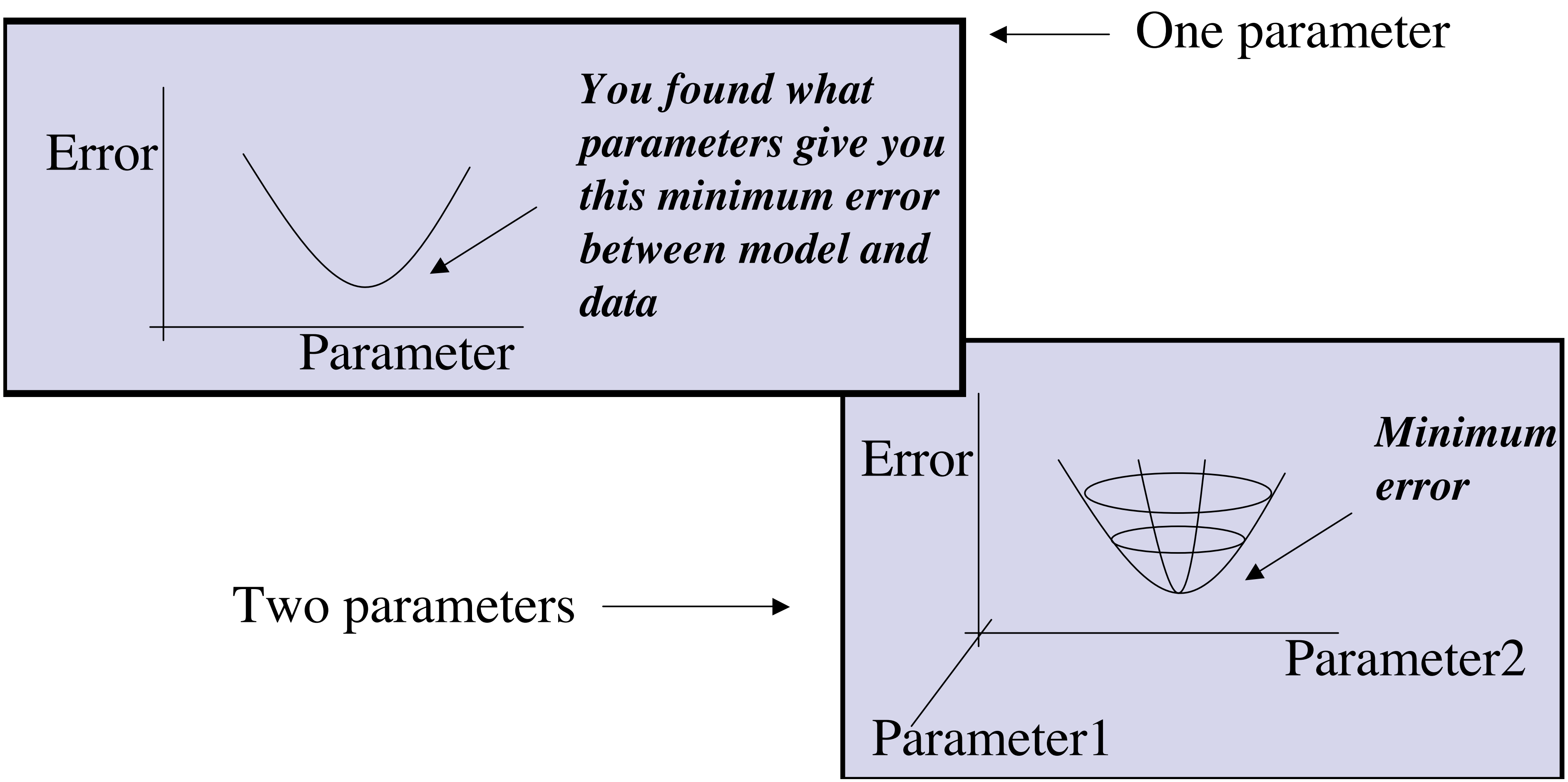
- There is a tremendous amount of interest in optimization and optimality in general in fields studying human cognition and behavior, such as Cognitive Science
  - **For model fitting in general**
  - **But also because it is intuitive to understand many aspects of human behavior in terms of optimization**

How does this relate to behavior and cognition?

- One popular model group used by cognitive science relates decision processes to minimization of cost and maximization of rewards (behaviorism)
  - **"I'm hungry, I need to eat" ->this hunger instinct and the dislike of discomfort leads us to make choices to minimize hunger, unless another cost/reward outweighs that choice**
  - **You drive on the correct side of the road because you don't want to have a head on collision with another car, or get a ticket because either of those would be a cost**
- Motor control (control of movement)
  - **Many aspects of human sensorimotor system are optimal in some sense (specifics vary, but examples are energy expenditure/recovery, time to goal, obstacle avoidance)**

You have already performed some optimization in this class

- Least squares
  - **However in that case you could compute the optimal point (which is the minimum of some error function)**
  - **In that case the cost function was a quadratic function (shaped like x^2), but it isn't always**
    - Sometimes there are many minima (we call those local minima)
    - It may be difficult to compute all the minima, or any for that matter


Error
Parameter
You found what parameters give you this minimum error between model and data
One parameter
Error
Parameter2
Parameter1
Minimum error
Two parameters

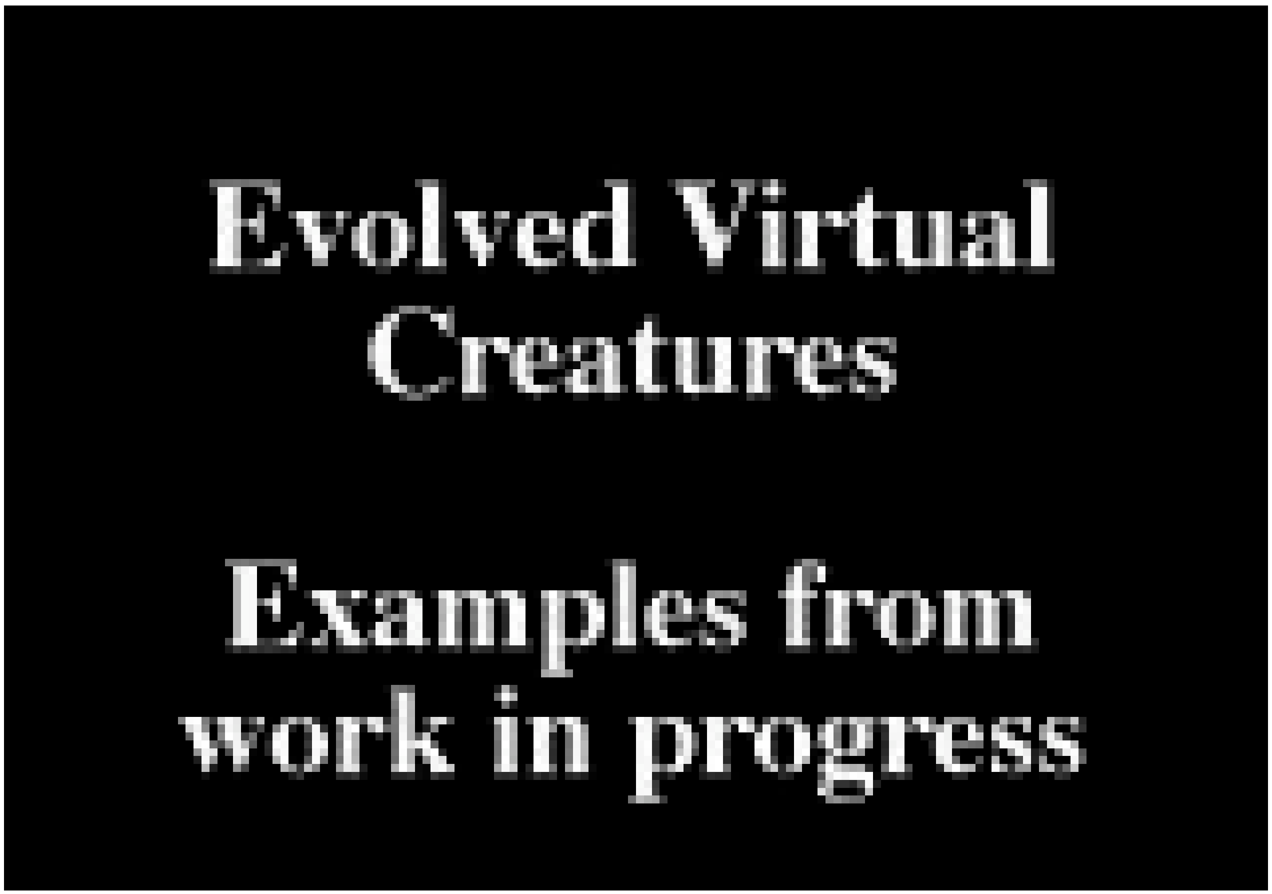
Today we'll discuss approximate solutions

- Works when you CAN'T easily solve the equations exactly (which is VERY frequent in nonlinear systems such as the brain, behavior, motor control, speech processing/synthesis/comprehension, perception, and more cognitively relevant topics)

Remind me again, what exactly are we ‘minimizing’ or ‘maximizing?’

- Minimize cost
- Maximize reward
- We decide what that function is
  - **Then have some unknown constants**
  - **Then we use these methods to find the constants**
  - **Those constants give us the smallest cost or largest reward function**
    - Can be then interpreted as the ‘best fit’ given a definition of what ‘goodness’ is

Graphical example - evolving organisms optimize cost, maximize rewards



What's one way to do this?

- Start with our simple question - how do we fit a model which is nonlinear in the parameters?

$$y = ax + e^{(bx)}$$

- We can use optimization methods to intelligently minimize the error between model and data

Nelder-mead simplex method

- Built into matlab
- Simple to implement
- How does it work?
  - http://www.boomer.org/c/p3/c11/c1106.html
  - Lagarias, J.C., J. A. Reeds, M. H. Wright, and P. E. Wright, "Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions," SIAM Journal of Optimization, Vol. 9 Number 1, pp. 112-147, 1998.