CogSci 109: Lecture 16

Wednesday Nov. 14, 2007
Nonlinear interpolation (Lagrange examples, Splines)

Outline for today

- Announcements
- Smooth interpolation for modeling cognitive and behavioral processes
- Lagrange examples
  - **Good use of lagrange**
  - **Bad use of lagrange**
- Splines
  - **Introduction**
  - **Examples**
  - **The mathematics**
  - **Matlab implementation**

Announcements

- Book sections on lagrange, splines, LERP, etc
- Homework due
- Midterm scantron portion
- Next assignment
  - **Readings**
  - **Short pre-break TBA**

Some applications for interpolation

- Filling out missing data points
- Matching data sampled at different rates
- Creating cognitive experiments
  - **Sensorimotor stimuli (i.e. tracking tasks)**
  - **Visual stimuli**
- Human beings typically behave in a smooth manner, rather than with changes so abrupt that change is instantaneous
  - **Interpolated curve fits to human movement can be used to compute quantities such as energy expenditure, optimality over some cost, etc**
  - **Linear interpolation is not optimal in terms of energy for example, since large accelerations require massive amounts of energy**

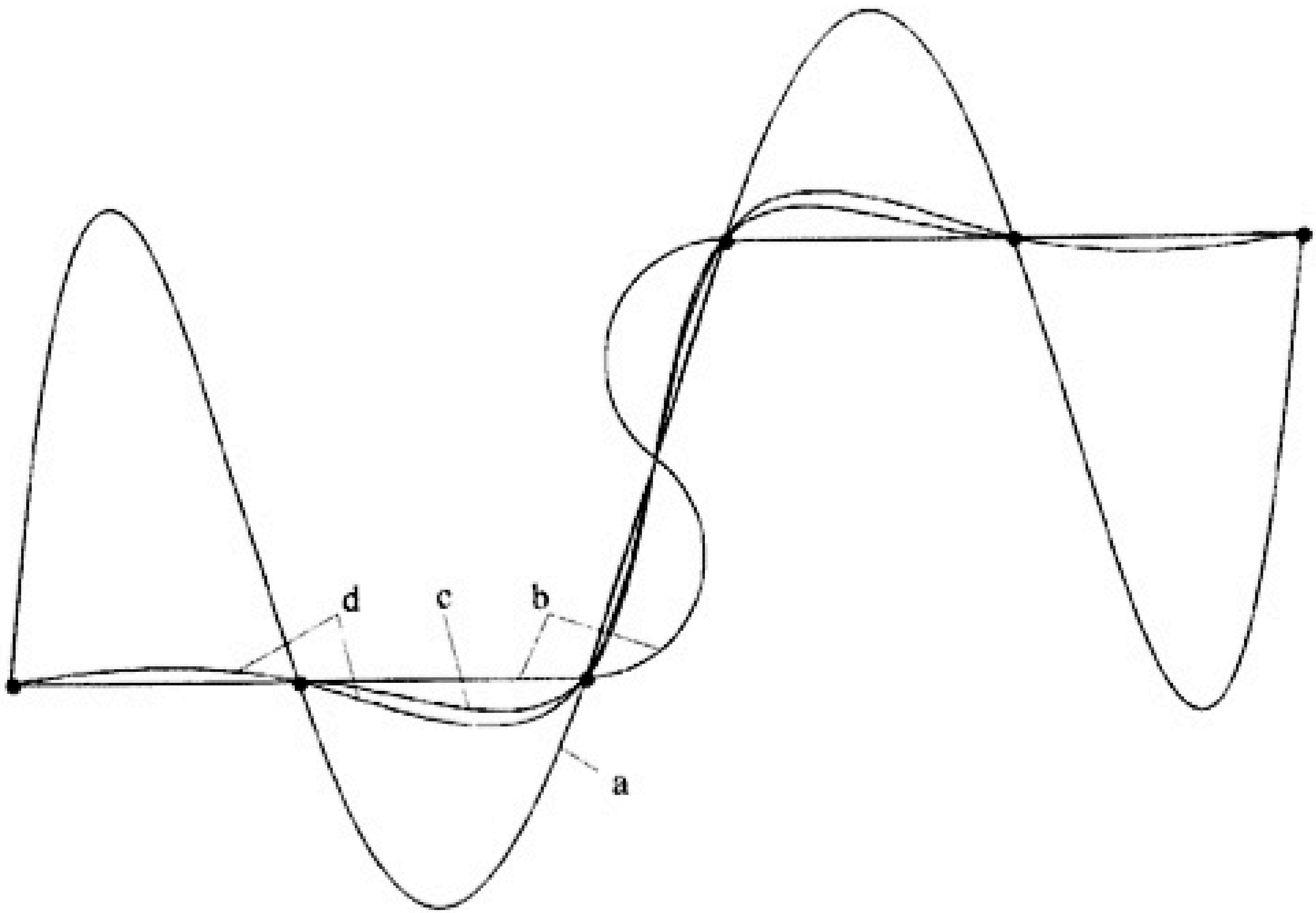


Figure 7.1 Interpolation curves drawn for six vertex points (dots), with y plotted in the vertical and x in the horizontal direction. Curves are shown for (a) a high-order polynomial fit, (b) a circular-arc fit, (c) a parabolic blend, and (d) a natural cubic spline.

A Good Lagrange example

- Fitting a small number of points
- <matlab>

An example of where not to use Lagrange

- Large number of data points
- <matlab>

So what can we do about that?

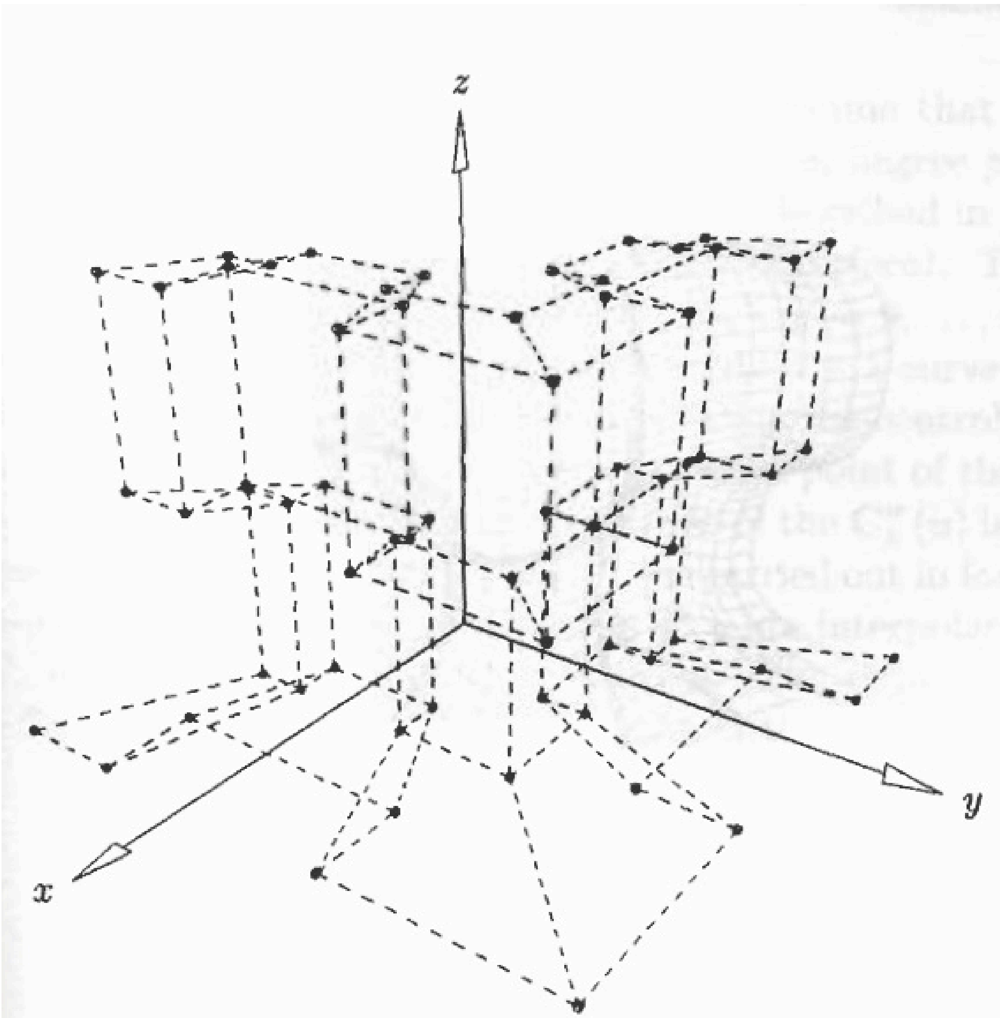
- Use only a small number of data points
- Use another method that doesn't have problems due to large numbers of data points
- What's one answer?

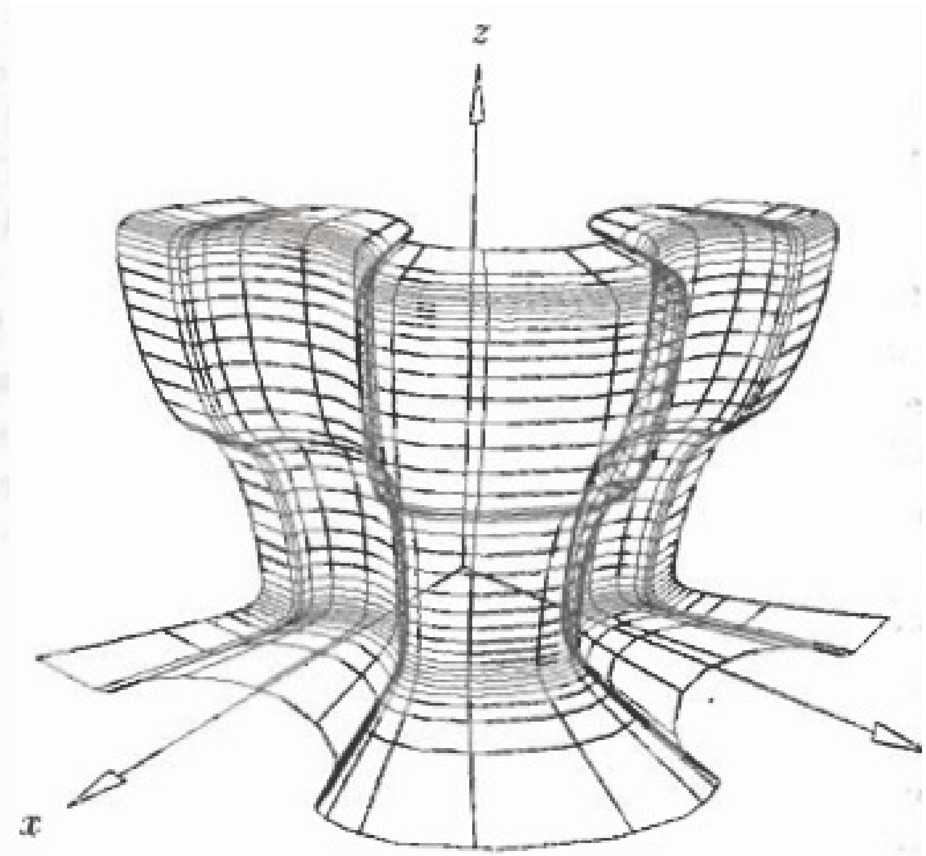
SPLINES

Splines are useful in many places Lagrange fails

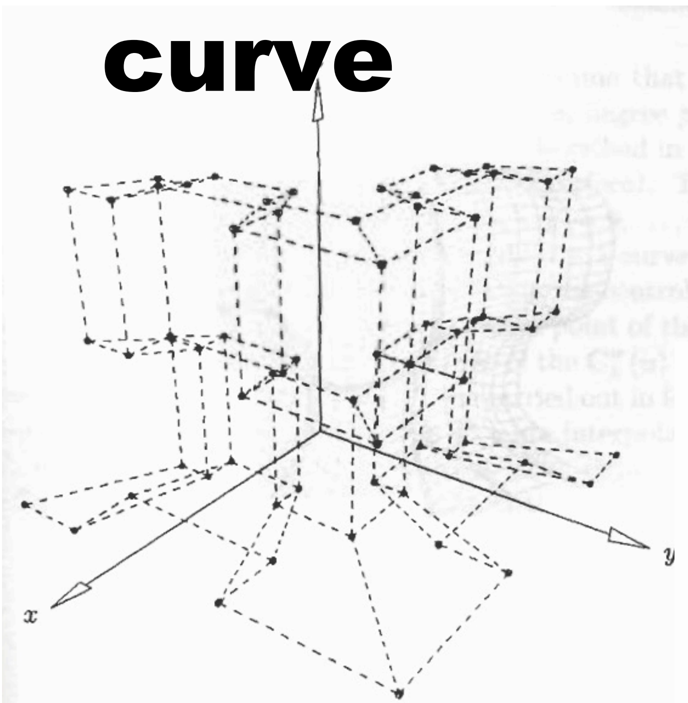
- Large number of data points
- Also can make a curve that passes through all data points
  - **some types do not enforce this**
- Drawn from drafting who drew from classical fine woodworking
  - **Thin piece of wood stretched between pegs to create curves**
  - **Many types of splines dependent on end conditions**
    - Pull tightly on the spline, curve gets sharper about the data points

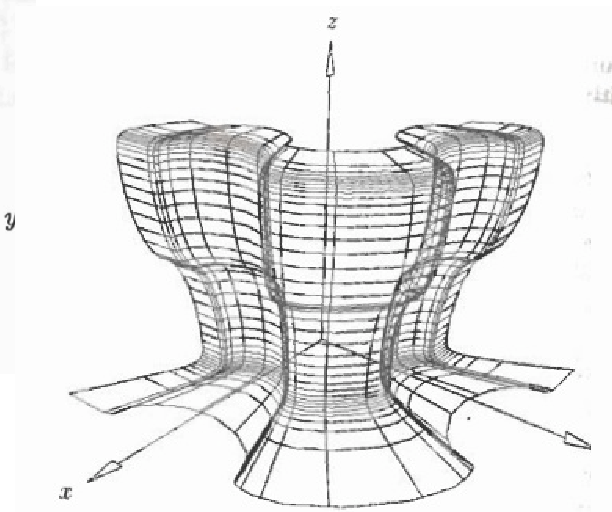
Splines are useful for N-Dimensions

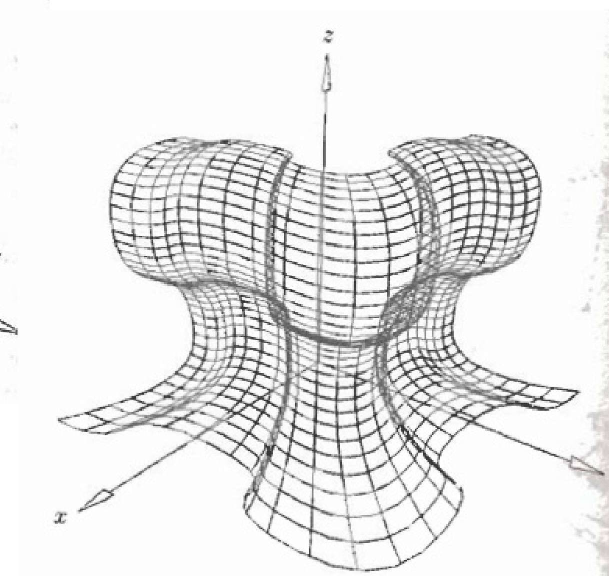




Splines also give you control over the final outcome of the curve







Some types of splines

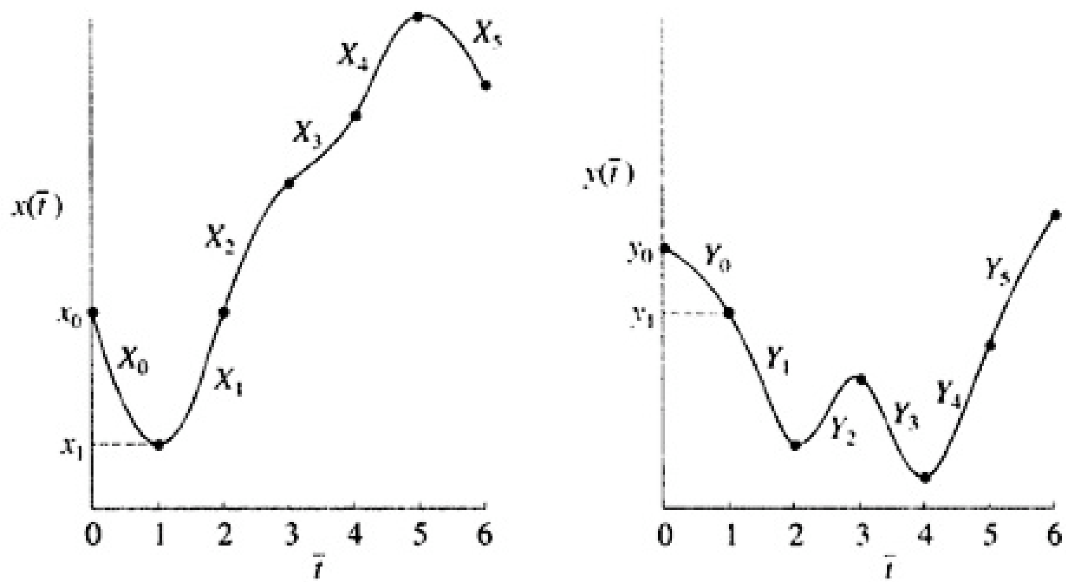
- Natural cubic spline
- Quadratic B-Splines
- Hermite Cubic Splines
- Coons Cubic Splines
- Rational B-Splines
- NURBS (Non-Uniform Rational B-Splines)

What we will discuss

- Natural cubic splines
  - **Why cubic?**
    - Because a curve is ‘wiggly’ and this is the lowest order polynomial that satisfies the conditions we’re going to lay out
    - Higher order gets too oscillatory

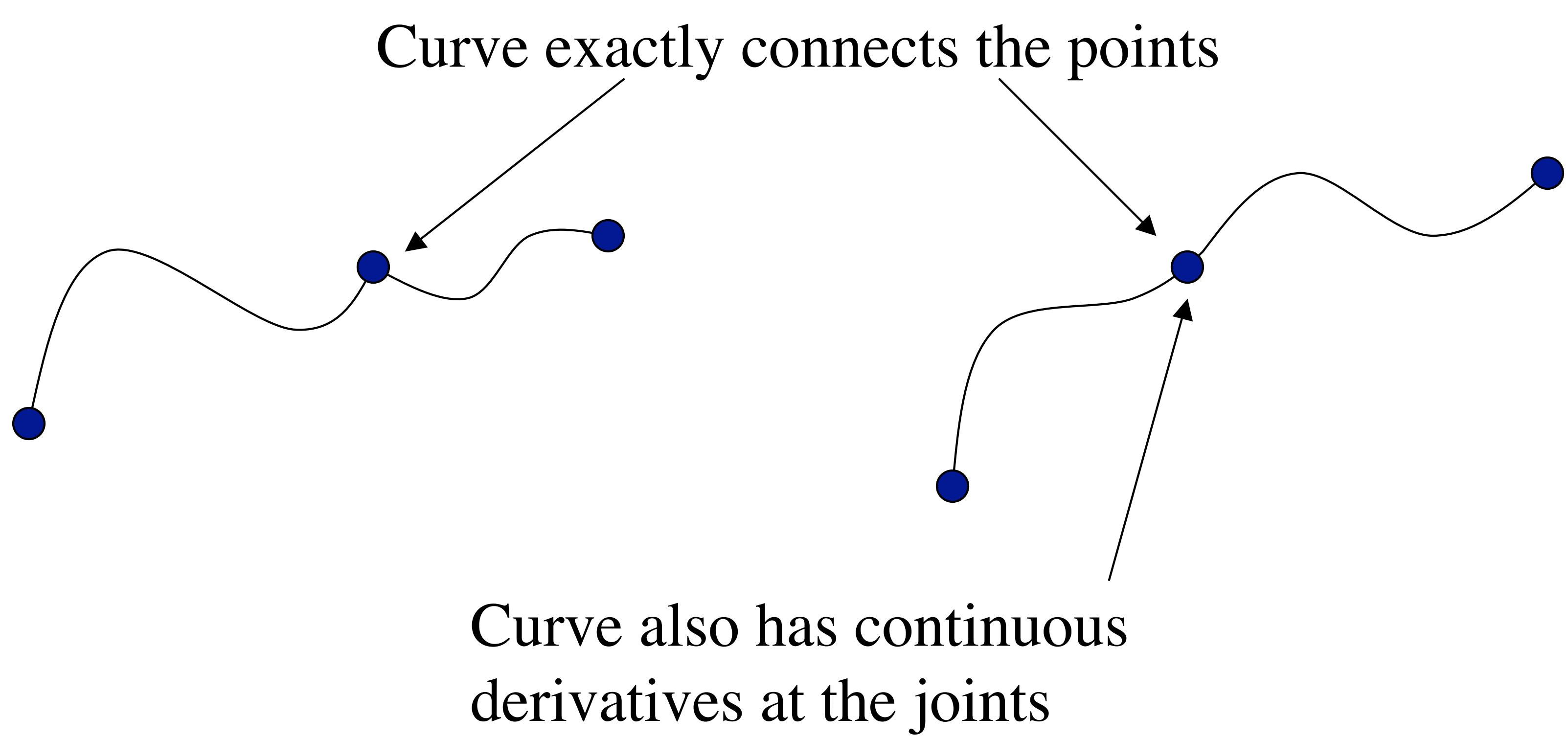
Natural Cubic Spline - a conceptual introduction

- We construct the following curves in sections



Adding constraints to solve for the unknowns

- Continuity at the joints:



Natural Cubic Splines

- We fit another parametric curve (similar to LERP), with a value of t from 0-1 again and make the ith segment according to

$$Y_i(t) = a_i + b_i t + c_i t^2 + d_i t^3$$

- And we solve for each set of these constants by requiring continuity at the end points (one section smoothly flows into the next, and the slope must match as well)

$$Y_i(0) = y_i = a_i$$

$$Y_i(1) = y_{i+1} = a_i + b_i + c_i + d_i$$

$$Y_i'(0) = D_i = b_i$$

$$Y_i'(1) = D_{i+1} = b_i + 2c_i + 3d_i$$