CogSci 109: Lecture 14

Monday Nov. 5, 2007
Linear interpolation (LERP, BERP, TERP, SLERP)

Outline for today

- Announcements
- What is interpolation?
  - **Definition**
  - **Applications, motivation for use**
  - **Orion nebula simulation**
- LERP - Linear interpolation
- BERP - Bilinear interpolation
- TERP - Trilinear interpolation
- SLERP - Spherical linear interpolation in polar coordinates
- Examples

Announcements

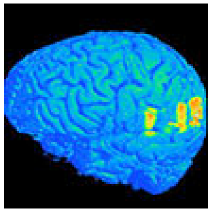
- Homework 4 is posted
  - **Due next Wed (Nov.14) because of Veteran's day**
- Homework 3 is almost done being graded
- No lecture Monday because of Veteran's day holiday!!!

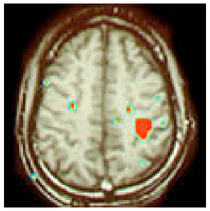
Announcements (II)

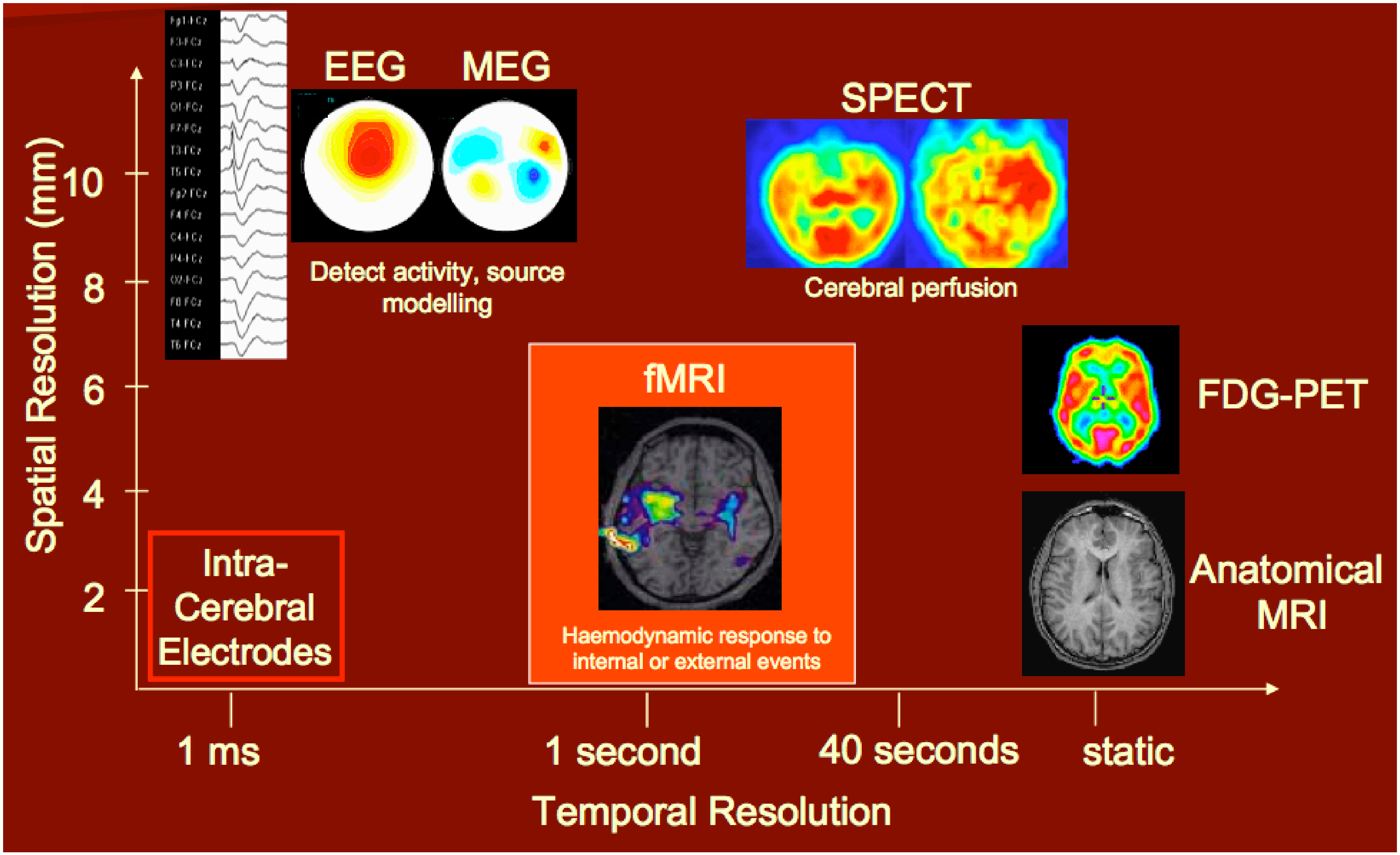
- Midterm Friday
  - **Midterm topics posted (last Fri)**
  - **Review session Wed 8pm location TBA**
  - **Practice midterm up later today/early tomorrow**
    - With solutions
    - Study and try to answer the practice without the solutions, then use the solutions to check yourself
    - Midterm will be similar in difficulty and topics/questions, so study carefully
  - **You should bring a calculator - any kind without wireless access, NO CELLPHONES OR PDAs**
  - **2 page, front and back (4 single sides) 8.5”x11” handwritten notes sheets are allowed**
  - **Bring a scantron**
  - **Bring a pencil**
  - **Part short answer, part mult. choice**

Consider again an old question

- Suppose I have just performed a brain imaging study where I recorded MRI *and* EEG data of subjects while they performed cognitive tasks *and* they had to perform a motor task (controlling a cursor with a joystick)
- The EEG is sampled at 2kHz (2000 samples/sec)
- The joystick and computer screen updates at 60Hz
- The MRI updates at approximately 1Hz
- How do I analyze this data in light of the fact that there are three very different sample rates?







EEG
MEG
Detect activity, source modelling
SPECT
Cerebral perfusion
fMRI
Haemodynamic response to internal or external events
FDG-PET
Anatomical MRI
Intra-Cerebral Electrodes
Spatial Resolution (mm)
10
8
6
4
2
1 ms
1 second
40 seconds
static
Temporal Resolution

Answering the question…

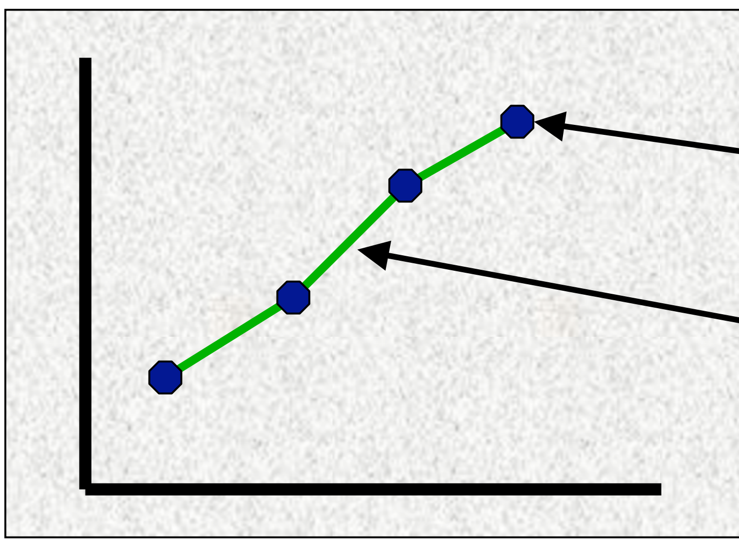
- I could *up-sample* the motor data
- But what if I want a smooth curve between points, and I know that the human does not inject significant disturbances between points?

•We don't want to do least squares because we want something to pass exactly through all data points!

•I could fit a curve that goes **through** all the data points

Interpolation defined

- Given a set of data points, we can construct a curve which fits exactly through each datapoint



Given this set of datapoints

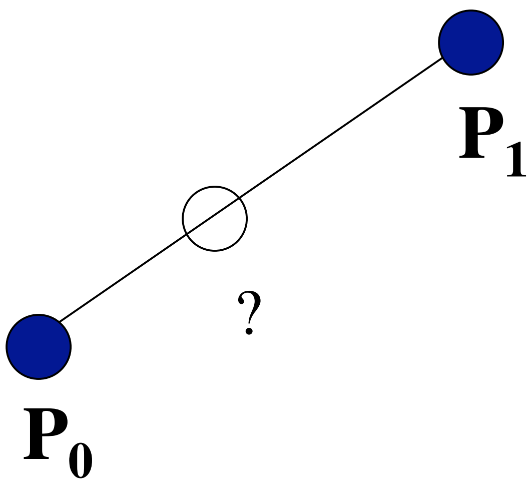
We want to fit a curve, or piecewise fit curves which pass exactly through each point

What can you do with this?

- Match data sampled at different rates
- Create surface plots with varying resolutions (mip-mapping for example with texture mapping)
- Create algorithms which store simplified representations of functions like sine, exponentials, etc (a lookup table), and when faced with points in between the stored values, the algorithm can interpolate between them
    - **Simplifies computation time**
- Create algorithms which take small amounts of data and interpolate to build models in an optimal way to make decisions based on

[redacted]

Linear interpolation ("LERP")



We use a parametric curve to blend between the two points:

$$P(t) = (1-t)P_0 + tP_1$$

Often this is written in the more efficient form:

$$P(t) = P_0 + t(P_1 - P_0)$$

There are less computations, only compute P_1-P_0 once per pair of points

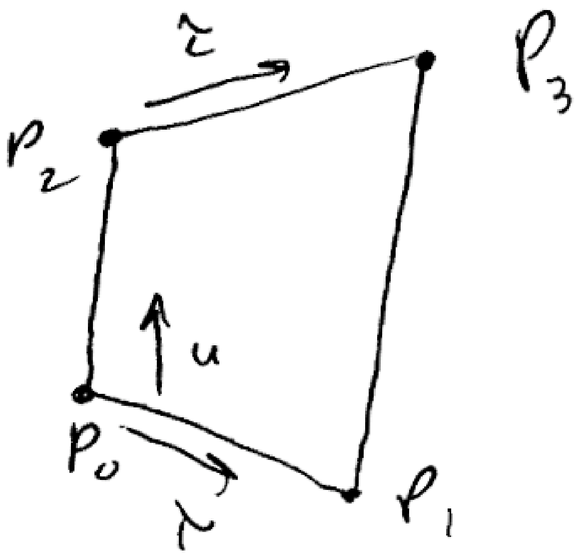
In 3D:

$$x(t) = (1-t)x_0 + tx_1$$

$$y(t) = (1-t)y_0 + ty_1$$

$$z(t) = (1-t)z_0 + tz_1$$

Bilinear interpolation ("BERP")



$$P_{01}(t) = (1-\tau)P_0 + \tau P_1$$

$$P_{23}(t) = (1-\tau)P_2 + \tau P_3$$

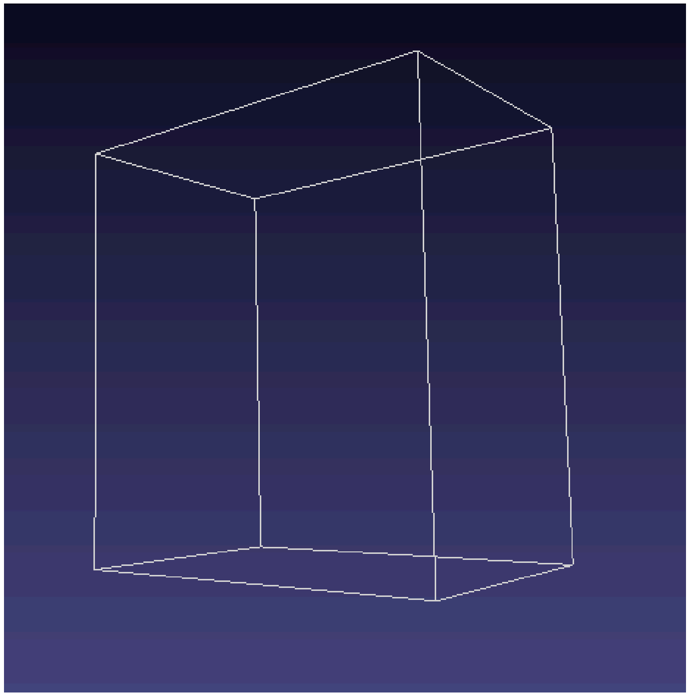
$$P_{0123}(t) = (1-u)P_{01} + uP_{23}$$

- Substituting the first two into the third:

$$P_{0123}(t) = (1-\tau)(1-u)P_0 + \tau(1-u)P_1 + (1-\tau)uP_2 + \tau uP_3$$

Thus given 4 points, we can find an interpolated point anywhere in the space between them

Trilinear interpolation ("TERP")



- How might you derive this such that we can interpolate to any point inside this cube?
  - **Same as LERP and BERP but a third interpolation parameter (another dimension)**

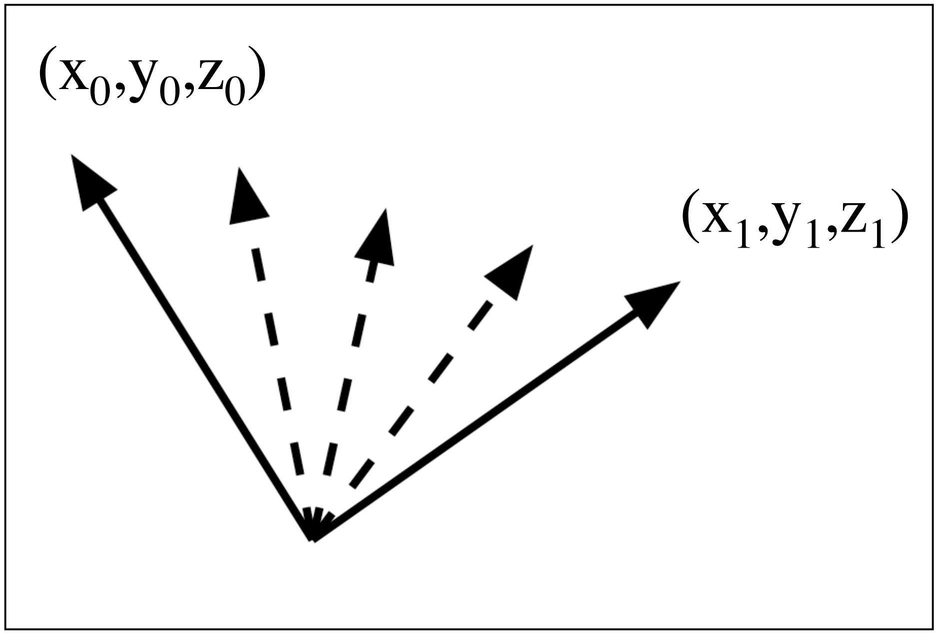
The points do NOT have to be evenly spaced

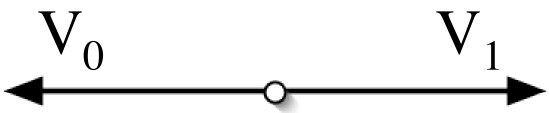
An important note about interpolating...

- By interpolating, you are not truly creating new data, you are blending between existing data
- Use with caution, since significant things might be happening between sample points if your data is too spread apart

Spherical linear interpolation("SLERP")

- Let's say we have two vectors we want to interpolate between:

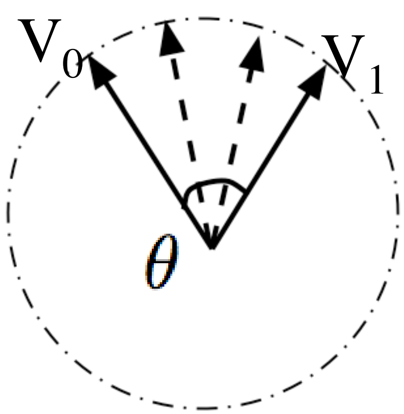




In case the angle = π (180°) we get a zero length set of vectors by our other interpolation method!

SLERP continued

- We need a circle in 2D with the center at the origin that passes through both vectors, and we'll interpolate over angle between the vectors, not length of the vectors



$$cos(\theta) = \frac{V_0 \cdot V_1}{||V_0||||V_1||}$$

This is from the definition of dot products

$$cos^{-1}\Big(cos(\theta)\Big) = cos^{-1}\Big(\frac{V_0 \cdot V_1}{||V_0||||V_1||}\Big)$$

Solve for theta by taking inverse cosine of both sides, and noting:

$$cos^{-1}\Big(cos(\theta)\Big) = \theta$$

$$\theta = cos^{-1}\left(\frac{V_0 \cdot V_1}{||V_0||||V_1||}\right)$$

SLERP (II)

- Now we can use the same interpolation equation, but with angles…we need to use a sine of the angle we're interpolating, since we're rotating about a circle:

$$\bar{V}(\tau) = \frac{sin[(1-\tau)\theta]}{sin(\theta)}\bar{V}_0 + \frac{sin[\tau\theta]}{sin(\theta)}\bar{V}_1$$

- Now as Tau goes from 0-1, our vectors are interpolated from V_0 to V_1
- So we compute the angle between the vectors by the dot product equation (with the cosine inverse from the prev. slide)

Example in matlab

- Shading interp