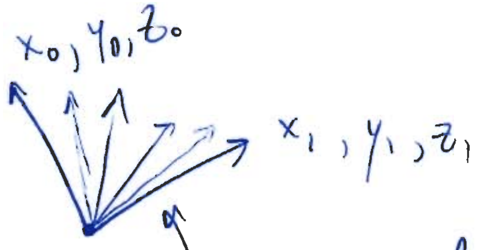
Spherical Interpolation ("SLERP")

- we want to interpolate vectors

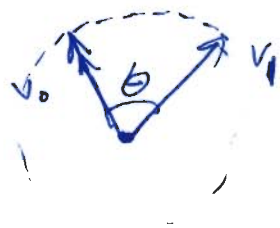


get wrong lengths using previous approach

$(y_1 = (1-t)x_0 + t y_1$

got 0 length vectors in case $\theta = \pi$!

we need a circle (in 2D) with center at origin that passes through both vectors



$$\cos\theta = \frac{\bar{V}_0 \cdot \bar{V}_1}{\|V_0\| \, \|V_1\|}$$

Find θ by taking $\cos^{-1}(\ )$ of both sides:

if not unit vectors, make them!

$$\theta = \cos^{-1}\left(\frac{\bar{V}_0 \cdot \bar{V}_1}{\|V_0\| \, \|V_1\|}\right)$$

then

$$\bar{V}(\tau) = \frac{\sin[(1-\tau)\theta]}{\sin\theta}\bar{V}_0 + \frac{\sin(\tau\theta)}{\sin\theta}\bar{V}_1$$

- we start at $\bar{V}_0$, end at $\bar{V}_1$

- it's all about perspective

Lagrange interpolation

passes polynomial of lowest poss. degree through all data pts.

- n data points → n parameters, degree is $(n-1)$

(1) $$f(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1 x + a_0$$

(2) $$f(x_i) = y_i \quad , \; i = 1, 2, \dots n$$

substitute (1) into (2), get

$$a_{n-1}x_i^{n-1} + a_{n-2}x_i^{n-2} + \dots + a_i x_i + a_0 = y_i \quad , \; i = 1, 2 \dots n$$

n algebraic eq. in n unknowns $a_0, a_1, \dots a_{n-1}$ since

x_i & y_i are known.

Could solve w/ Gaussian elimination, etc.

But usually the eq. become ill-cond. $n > \sim 5$

so it's hard to solve them accurately.

(review ill-conditioning)

another approach to solve for $a_0, a_1, \dots a_{n-1}$ where they must be linear comb. of y_i: ②

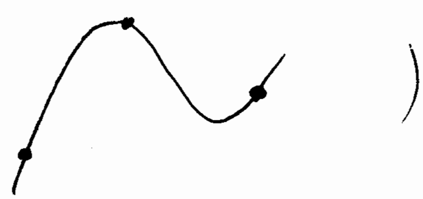
(3) $$f(x) = \sum_{k=1}^{n} y_k L_k(x)$$

L_k are polynomials of degree $n-1$

Recall a polynomial is given by

(4) $$P(x) = \sum_{k=0}^{n} a_k x^k$$

and we want the polynomial to equal the data point at those points (ie:



Thus we can make $L_j(x_i) = \delta_{ij}$

$i = 1, 2, \dots n$

where $\delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$

From algebra we recall that any polynomial of degree n can be factored into a const. multiple of a product of n factors $(x - x_\ell)$ where x_ℓ are ~~the~~ zeros of ~~the~~ polynomial since $L_j(x_i)$ is a polynomial degree $n-1$ (3)

with known zeros $(i \neq j)$, it has this form;

$$L_j(x) = C_j\,(x-x_1)\cdot(x-x_2)\cdots(x-x_n)$$

find C_j where $i=j$, so $L_i(x_j)=1$

$$1 = C_j\,(x_j-x_1)(x_j-x_2)\cdots(x_j-x_n)$$

$$C_j = \frac{1}{(x_j-x_1)(x_j-x_2)\cdots(x_j-x_n)}$$

thus L_j is :

$$L_j(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_j-x_1)(x_j-x_2)\cdots(x_j-x_n)}$$

$$\boxed{L_i(x) = \prod_{j=0,\, j\neq i}^{n-1} \frac{(x-x_j)}{(x_i-x_j)} \qquad y(x) = \sum_{i=0}^{n-1} L_i(x)\, f(x_i)}$$

reduces to $y_i = f(x_i)$ at all i

vertex points.